Staggered Rollout For Innovation Adoption*

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Abstract

I consider a mechanism design approach to innovation adoption and show how it is optimal for the principal to induce artificial scarcity to speed it up. Take-up of a new product generates information about its value for others, so agents want to free-ride before irreversibly adopting it themselves. This causes a time-delay externality that a principal seeking to achieve an adoption target as quickly as possible (for example, a government trying to reach herd immunity through vaccination while agents are uncertain of their personal vaccination benefits, not internalizing the positive externality of reaching the adoption target) seeks to avoid. Scarcity speeds up learning because it limits free-riding. I show that the possibility of imposing supply restrictions is always beneficial compared to free supply. I also show that optimal supply plans are simple in that there is a batched supply release with fewer batches than agents' value types. I fully characterize such optimal plans for settings with up to three types and show that (non-optimal) supply plans may be Pareto improving.

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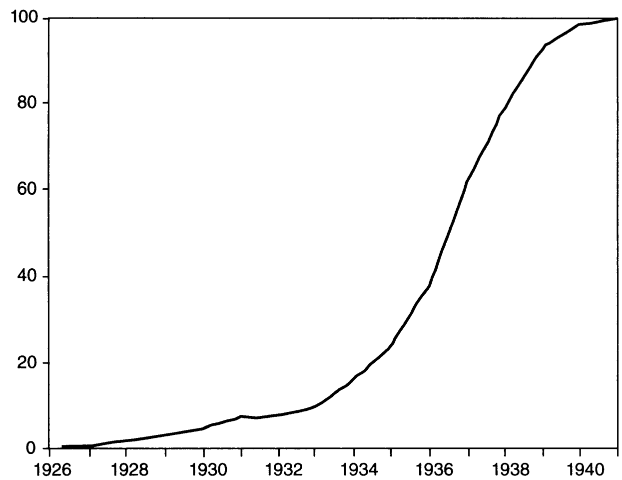
I Introduction

What affects the speed of technology adoption? Can it be easily sped up through the control of product availability? One of the most consistent regularities found in cases of innovation diffusion is the S-shaped format of adoption curves. That is, take-up at first increases convexly, and after a certain point, the growth rate decreases, and the curve becomes concave. An example from a highly influential work by Ryan and Gross (1943) is the adoption of hybrid corn by Iowa farms in the 1920s and 1930s:

Figure 1: Example of S-shaped adoption curve



Percent	0.0	0.4	1.4	3.0	4.6	6.9	7.3	9.6	15.8	24.0	38.0	62.0	79.0	93.0	98.5	99.7
Year	1926	1927	1928	1929	1930	1931	1932	1933	1934	1935	1936	1937	1938	1939	1940	1941

FIGURE 5. PERCENT OF ADOPTERS OF HYBRID CORN IN TWO IOWA COMMUNITIES, 1926–1941
(*From Ryan and Gross 1943, Figure 4*)

This adoption pattern is widespread in diffusion studies. In his influential book, Rogers Everett (2003a) goes as far as to say that:

"The S-curve of diffusion is so ubiquitous that students of diffusion often expect every innovation to be adopted over time in an S-shaped pattern".

One important reason why lags in the adoption of a new product of unknown value are observed is social learning: the adoption of a good by others generates information on quality that other agents can gather, but only over time. The literature empirically documenting social learning, especially for health and agricultural innovations, is large.[1] Ryan and Gross (1943), cited above, present survey evidence that

[1] For developing countries, see Besley and Case (1993); Besley, Case et al. (1994); Foster and Rosenzweig (1995), as well as Kremer and Miguel (2007); Conley and Udry (2010); Dupas (2014). For evidence of forward-looking behavior, see Munshi (2004); Bandiera and Rasul (2006).

the most influential factor in farmers' decision on whether to implement hybrid corn was the previous adoption of the technique by their neighbors.

I consider a model with a continuum of agents who can irreversibly adopt a good of unknown binary quality. They have heterogeneous adoption values that are always positive in the good state and negative in the bad. The set of values in the good state is assumed to be finite and characterize agent types. Potential adopters learn about the quality of the good through a perfect bad news Poisson process, with an arrival rate proportional to the number of adopters. In particular, if no news arrives, they become more confident that the product is high quality.

With no restrictions on adoption, there is a unique equilibrium outcome of the game, called a free supply path. Importantly, it features strategic delay, with agents choosing to adopt later even when adoption would yield positive expected utility. This happens because individuals want to learn from the experience of early adopters, which can be inefficient even from the perspective of agents' total welfare.

My first result is that with free supply, this model can generate an S-shaped adoption pattern, which the literature could only generate by imposing myopic agents or adoption restrictions.

I then investigate how adoption and learning can be sped up. I consider a supplier who wants to reach a target adoption rate quickly. Regardless of the state, she wants to reach an adoption target as soon as possible. This objective is plausible in many settings. Examples include a government trying to reach mass immunity before new virus mutations come by, a matchmaker (or service) platform wanting to reach a critical mass of adopters as soon as possible on one side of the market (Evans and Schmalensee, 2016) or a new product for pest control by farmers (Reeves, Ohtsuki, and Fukui, 2017). In general, new innovative products in different sectors must often show their profitability to managers and company shareholders as soon as possible.

My second result is that the principal can always improve on free supply. The product can be released through batches, with any unclaimed units available for take-up as long as others have not claimed them. The principal can reach an adoption target earlier by simply never supplying more than this threshold level. Given competition for the mass available, agents of the lowest value type exhaust the unclaimed supply earlier on, with all applying to get the good, which is then rationed to part of the applicants.

Optimal supply plans with one type have a strikingly simple formulation: add one batch at the start, with just enough to reach the target adoption rate. Competition for available units makes all equilibria with informational free-riding unravel, which leaves only one possible equilibrium, with all agents applying to get the good from the start.

Interestingly, when considering a general setting with heterogeneous types, it is the case that "simple" supply plans are optimal for the principal, with only the release of at most as many batches as types. These batches are also released at points in time in which the demand for just-released units is weakly higher than the supply. The reasoning is straightforward: if any other plan were to be strictly better than all in this class, it must lead to earlier adoption for the lower value types. But then this would mean that some agents would not be willing to adopt the innovation before this final moment, delaying learning.

Additional insights are gained by focusing on cases with a particular number of types. Consider a model with high and low-value agents. Instead of threatening not to resupply the good, the principal commits to supplying it with delay. The tension (from the principal's perspective) comes from the fact that competition generates earlier and more "learning" for agents less willing to adopt the good. However, this can only work if there are no incentives for higher types to also free-ride and adopt later. The main point is that given a choice between taking up at the initial moment and the final one, with the less willing agents, high types must prefer the former if scarcity is to benefit the principal.

With three types, the optimal plan is such that earlier adoption from higher types is *more valuable* for the principal than later adoption of mid-level valuation types. This, in turn, is a consequence that adoption is easier for more willing types (those with a higher valuation) and preferred by the principal because the whole stock of adoption is used for changing beliefs. Therefore, a positive mass of earlier adopters will lead to more learning over time than later adopters. Interestingly, if at least two types are willing to adopt the good myopically, the principal can make Pareto improvements by introducing supply restrictions.

As one considers settings with an arbitrarily large number of types in a bounded interval, approximating a continuous distribution with full support, the number of batches in an optimal plan eventually becomes strictly lower than the number of types for any such distribution. This suggests that optimal plans are not only simple, consisting of only as many batches as there are types, but also that this number is strictly lower than a certain proportion of the number of types.

Finally, I discuss some extensions of the model for different principal objective functions. I also discuss how a principal can make welfare gains for all agents through supply restrictions. I then consider a case of two regions differing only on the arrival rate of news and show that the principal should still aim to reach a target adoption simultaneously. Finally, I show that the possibility of having a second product, which would potentially have the benefit of ensuring the principal against the possibility of having one product revealed to be bad may not benefit her, as it would hamper the effect of adding scarcity at the final moment.

Related Literature

This paper relates to several social learning literature branches, which have used different assumptions about adoption and different objectives for a planner (or principal). It is also the case that, unlike the previous literature, the present paper can generate the widely empirically documented S-shaped adoption curve with free adoption and without imposing myopia on the agents.

Most closely related to the present setting are Frick and Ishii (2023), Laiho and Salmi (2021), and

Laiho, Murto, and Salmi (2022). Frick and Ishii (2023) consider a model with perfect bad news and find that the unique equilibrium involves convex adoption. Laiho and Salmi (2021) consider a model with perfect bad and good news and discrete time and a monopolist interested in maximizing profit. Laiho, Murto, and Salmi (2022) study how dynamic posted prices can achieve the social optimum in a setting with Brownian signals.

Some papers also model social learning when signals of quality depend on the adoption decisions of others. Young (2009) focuses on parsing out adoption from mimicking and social learning through the shape of the adoption curve, assuming that agents are myopic about their dynamic incentives to delay adoption. Perego and Yuksel (2016) analyze a model in which network-connected agents can learn from their own experience or others, aggregating information, and show that increasing connectivity may decrease information quality. Wolitzky (2018) studies a model with observable outcomes of a random sample of players but not their actions. It shows that inefficiencies can persist as the sample size goes to infinity and even increase with it.

Other papers have considered artificial scarcity as a tool a principal may use to achieve its objectives. In a recent paper, Parakhonyak and Vikander (2023) show that artificial scarcity may be a quality signal if players observe only overdemand and may infer high quality by assuming that others observed high private signal realizations. Unlike in the present paper, scarcity is a signaling tool that induces faster take-up, rather than through competition effects. DeGraba (1995); Nocke and Peitz (2007); Möller and Watanabe (2010) are other examples of papers considering beneficial supply shortages by their hampering effect on strategic delays, but do not deal with social learning, environments, only exogenous learning of private valuations over time.

Bonatti (2011) and Che and Mierendorff (2019) are two papers of note. They consider monopolists and social planners with one or many goods of different but unknown quality. Similarly to Laiho and Salmi (2018), Bonatti (2011) studies a monopoly firm of a durable good that uses dynamic prices to maximize revenue. Che and Mierendorff (2019) study recommender systems of goods facing short-lived customers (who, therefore, cannot strategically delay adoption). A social planner recommends a good product to some, but not all agents, in the absence of perfectly good news, as committing to recommending it to all agents leads to a less informative recommendation. Importantly, Bonatti (2011) considers a setting in which actions are reversible, which leads to a significantly different optimal strategy for the principal.

The papers above show diverse reasons for a planner or monopolist to restrict access to goods to increase profits or welfare. I contribute to this literature by showing that competition effects may be used to reach adoption targets faster in settings where signals of quality may depend on aggregate adoption for reasons that are different from profit maximization, which is not a good representation of the objectives of suppliers of innovative goods in many cases.

Another related literature is the one on herding models, which assumes that individuals observe the actions taken by those who come before them but not signals of quality depending on these actions. It is common for this literature to be referred to as "social learning" literature. Still, our settings are substantially different, although related in that the previous decisions of others matter for each agent. Examples from this vast literature are (Scharfstein and Stein, 1990; Banerjee, 1992; Bikhchandani, Hirshleifer, and Welch, 1992; Smith and Sørensen, 2000; Eyster and Rabin, 2014; Smith, Sørensen, and Tian, 2021).

There is substantial and influential literature involving many other fields of study besides economics on the diffusion of innovations. As mentioned above, a good review can be seen on Young (2009). Another famous reference is written by one of the founders of the diffusion field, Rogers Everett (2003b). As mentioned before, almost all these papers assume that agents myopically adopt the innovation.

Lastly, part of the diffusion literature focuses on social learning in networks. This includes (Jackson, 2010; Mossel, Sly, and Tamuz, 2015; Akbarpour and Jackson, 2018) and has been recently reviewed by Golub and Sadler (2017). The focus here is on network structure, so the forward-looking behavior of agents is generally not assumed. S-shaped adoption curves are also observed in much of this literature.

This paper is structured as follows: section 2 presents the model setup; section 3 will discuss the dynamics with free availability; section 4 will then discuss how supply restrictions can help the principal; and finally, section 5 discusses the results from the previous section and considers extensions.

II Model

II.A Actions and Payoffs

Consider a population composed of a continuum of agents I. There is a measure η over I in the Borel σ-algebra of the product-topology of I. I normalize $\eta(I)=1$. A superscript i will be used for a generic agent. They can apply to get a good, and are out of the game after a successful application. I will use the terms "take up the good", "adopt the innovation", or simply "adopt" interchangeably. Denote agent i's action at time t by $a_t^i \in \{0,1\}$. If $a_t^i = 1$, an agent applies to get the good. At each point in time, agents are able to see a private history of previous applications and the realization of a public signal depending on the stock of adopters, as will be made clear later on. There is a persistent state of the world $\omega \in \{b,g\}$ (also referred to henceforth as "bad" and "good" states, respectively) with a probability $p(\omega) \in (0,1)$ of a ω realization.

All agents receive the same payoff from adopting if the state is b, which I normalize to -1. If the state is g, however, agents receive type-dependent payoffs. To ease notation, I will denote an agent type by her g-state payoff $v \in V$. The set V is assumed to be finite, with elements (v^n,q^n), with the first argument representing payoffs v^n with $n \in \mathcal{N} \equiv \{1,2,...,N\}$, and $v^n > v^{n+1}$ for any $n < N$. I also have that $v^n > 0$ for each n, which means that all types would want to adopt in the good state of the world. The entire vector of valuations is denoted by $v^{\mathcal{N}} \equiv (v^1,...,v^N)$. Agents get a flow payoff of 0 at each moment when they do not adopt. The mass of agents with valuation v^n for each $n \in \mathcal{N}$ is given by q^n, with $\sum q^n_{n\in\mathcal{N}} = 1$. The vector representing the mass of agents with each valuation is represented by $q^{\mathcal{N}} \equiv (q^1,...,q^N)$. Despite the fact that a type is a couple $(v^n, q^n) \in V$, to ease notation, I will instead say that agents are of type v^n. Agents discount future payoffs exponentially with a common discount rate $r > 0$.

II.B Social Learning

Agents learn about the state of the world through a public signal. A public signal process better represents the settings that motivate this paper, such as vaccination and pest control. I focus on **perfect bad news Poisson public signals**, which are frequently studied by the strategic experimentation literature (e.g. Keller and Rady (2015)). No signal realization can ever happen if the state is good. If it is bad, however, a positive arrival rate of a perfectly informative signal reveals to all that the state is b. When a realization occurs, I say that a *breakdown* happened, borrowing the term from the literature.

Crucially, previous adoption decisions of others influence the arrival rate of news. This means that learning happens socially, but not only through the observation of adoption decisions of others, as in herding models[2]. Formally, the arrival rate of bad news at time t depends on the mass of adopters up to that point in time, M_t, absent realizations up to that point. The arrival rate at time t is given by $\beta M_t dt$, with $\beta > 0$. I denote by $M^{\emptyset}$ the **adoption path** $\{M_t\}$ such that $M_t = 0\ \forall t \geq 0$, and the set of all adoption paths $\mathcal{M}$.

The common prior that $\omega = g$ is representeed by $\mu_0 = p(g) \in (0,1)$. The posterior, *absent a breakdown*, μ_t, is determined by Bayes' Rule:

$$\mu_t = \frac{\mu_0}{\mu_0 + (1-\mu_0)e^{-\int_0^t \beta M_\tau d\tau}}$$

As described above, if a realization of the signal happens at a time t, μ_t discontinuously goes to 0, and all agents learn that the state is bad. Given the payoffs described above, no agent ever adopts the innovation from this point forward.

Note that in this setting, as for any adoption path that is not $M^{\emptyset}$, $\{\mu_t\}$ is strictly increasing in t and $\lim_{t\to\infty} \mu_t = 1$. This is the case as if no bad news arrives, agents become strictly more optimistic about the state of the world, arbitrarily so over time. If no mass adopts the good from time 0 to a time t, then no news can arrive, and $\mu_t = \mu_0$. Note that even if a positive mass adopts exactly at time t, it is still the case that $\mu_t = \mu_0$, as beliefs can only change over time, so that for any $t' > t$, $\mu_{t'} > \mu_t$.

It is worth pointing out that the above model, with heterogeneous payoffs in the good state and a common prior, can also be modified to represent settings in which the value of the innovation is

[2]Note that another difference between usual herding models and the one of this paper is that the former has a queue for adoption, with each agent being able to see only the decisions of the others who came before her

homogeneous but priors are heterogeneous. Essentially, both capture that individuals have a "threshold" belief above which agents decide to adopt the innovation.[3]

II.C Principal

A principal, who also does not know the state of the world and shares the same prior μ_0 with the agents, wants to reach a target $\bar{M} \leq 1$ adoption rate as soon as possible. Formally, her flow payoff is given by 1 if $\bar{M}$ is reached, 0 otherwise. This means that she commits to a supply plan maximizing $\int_0^\infty e^{-rt} Prob(M_t = \bar{M}) dt$.

The principal cannot make transfers to agents and has, as her only tool to achieve faster take-up, the ability to commit to the release of supply over time. Formally, a **supply plan** is a step-function $S : [0,\infty) \to [0,1]$. The principal is constrained to release new supply only through batches. A supply plan is defined below:

Definition 1. *A **supply plan** is a function S defined for each time $t \in [0,\infty)$ that is:*

1. *Right-continuous, so that every $t \geq 0$, $\lim_{\tau \to t^+} S_\tau = S_t$.*
2. *Non-decreasing, so that if $t' > t$, then $S_{t'} \geq S_t$.*
3. *A step-function up to $M \in \mathbb{N}$ points: there can be at most M times $t_1, t_2, ... t_M$ in which $S^-_{t_i} \equiv \lim_{t \to t_i^-} S_{t_i} < S_{t_i}$, for $i \in \{1, 2, ..., M\}$, with $M > N$.*

The first point precludes the possibility of releasing the good anytime after a certain point, but not at that exact moment. The second one restricts the principal to increasing supply, never burning stock. The third one tells us that the principal can only add a finite number of batches. Note that M is only restricted to be above the number of types, but it can be arbitrarily large.

Denote the set of all supply plans by $\mathcal{S}$. I will consider different assumptions on the set of available plans for the principal. In particular, I will focus on a setting where no scarcity restrictions can be imposed (free supply), and only plans contingent on time, not history of adoption, but I will discuss this possibility later on in a .

II.D Applications

Agents can apply to get the good at each point in time. If the mass of applicants is lower or equal to the available supply, the entire mass of agents gets it. Otherwise, the available mass is rationed among the applicants. Define the mass of applicants at a point in time t by $A_t = \eta\{i \in I | a^i_t = 1\}$. The sequence $\{A_t\}$ will be referred to as an **application path**. Denote $\lim_{t \to t^-} M_t$ by M^-_t. For any t, if $A_t \leq S_t - M^-_t$,

[3] Thresholds beliefs for adoption are widely used in adoption studies. For example, the network adoption literature cited in the Related Literature I section often considers models with this feature.

applicants will receive the good with probability 1. I will assume that if $A_t < S_t - M_t^-$, all applicants will receive the good (not only the entire mass, but every agent), but if $A_t = S_t - M_t^-$, potentially a mass zero of agents might not successfully adopt. If, instead, $A_t > S_t - M_t^-$, a mass equal to $S_t - M_t^-$ will receive the good, while the rest is still in the game. This implies that there is a conditional probability of success (if the agent applies at t) of $(S_t - M_t^-)/A_t$ if $A_t > 0$. Otherwise, if $A_t = 0$, we set the probability of a successful application to 0. If $M_t^- = S_t^- = S_t$, the supply is exhausted, and no players can get the good if they apply, until a new supply mass is released. Define the **acceptance probability path**, conditional on aggregate application behavior $\{A_t\}$, supply plan $\{S_t\}$ and application at time t by $\{Q_t^i\}$. It is defined as follows: :

Definition 2. *Given a supply plan $\{S_t\}$, adoption path $\{M_t\}$ and application path $\{A_t\}$, the* ***acceptance probability path*** *for agent i is denoted by $\{Q_t^i\}$, defines a probability of succesfull application at time t:*

$$Q_t^i = \begin{cases} 0, \text{ if } M_t^- = S_t^- = S_t \\ 0, \text{ if } S_t < M_t^- + A_t \text{ and } A_t = 0 \\ (S_t - M_t^-)/A_t, \text{ if } S_t < M_t^- + A_t \text{ and } A_t > 0 \\ 1, \text{ otherwise} \end{cases}$$

Each adoption path that I will consider induces a non-empty set of times $\mathcal{T}^{a^i}$ in which agent i applies to adopt with $Q_t^i > 0$, absent a breakdown up to this point in time for any path $\{M_t\} \neq M^{\emptyset}$. Non-emptiness comes from the point above that $\mu_t \to 1$ in that case and the structure of payoffs (0 from not adopting, bounded value v if adopting when the state is bad). It is clearly the case that $Q_t \in (0,1)$ for at most M times, one for each time in which $S_t^- < S_t$. Finally, we say that $o_t^i(a_t^i, A_t, S_t, M_t) = 1$ if an application by agent i at time t is successfull, and $o_t^i(a_t^i, A_t, S_t, M_t) = 0$ if either the agent did not apply at this point in time or had an unsuccessful application.

II.E Information Sets and Strategies

Information sets for an agent i are either $\emptyset$ (before any application by the agent) or composed of a finite sequence of up to $K \geq M$ unsuccessful applications. $h_i^{t_m} = ((t_1, U), (t_2, U), (t_3, U), ..., (t_M, U))$, M being the upper bound on the number of supply jumps. The set of all possible histories for agent i is $\mathcal{H}_i$.

Let $\hat{h}_i^{t_m}$ be the extension of $h_i^{t_m}$ which records the date at which i applied successfully if such a date exists, so it may take the form $\hat{h}_i^{t_m} = ((t_1, U), (t_2, U), (t_3, U), ..., (t_{M-1}, U), (t_M, S))$ where S denotes a successful application. Let $\hat{\mathcal{H}}_i$ denote the set of i's possible extended histories.

A belief for agent i at history $h_i^{t_m}$ with $m < K$ is as a function φ^i over $\hat{\mathcal{H}}_j$ for $j \neq i$, with support restricted to either $\emptyset$ or histories of the form $h_j^{t_l}$ with $t_l \leq t_m$, the private history of applications. Define an element of $\Pi_{j \neq i} \hat{\mathcal{H}}_j$ by $\hat{h}_{-i}^{t_m}$ and an element of $\Pi_{j \neq i} \mathcal{H}_j$ by $h_{-i}^{t_m}$ .We have, then, beliefs $\varphi^i(\hat{h}_{-i}^{t_m} | h_i^{t_m}, h_{-i}^{t_m})$.

The optimal strategy (never to adopt) is obvious whenever a signal realization comes by. Therefore, we can condition adoption strategies only on the other fundamentals of the model absent breakdowns.

Given a function φ^i, note that at every history, there is an induced expected in equilibrium $\{\tilde{M}_t\}$ and $\{\tilde{A}_t\}$ define a perceived acceptance probability path $\{\tilde{Q}^i_t\}$:

Definition 3. *For every $h^{t_m}_i \in \mathcal{H}_i$, $h^{t_m}_{-i} \in \Pi_{j\neq i}\mathcal{H}_j$, take the beliefs $\varphi^i(\hat{h}^{t_m}_{-i}|h^{t_m}_i, h^{t_m}_{-i})$. A **perceived acceptance probability path** $\{\tilde{Q}^i_t\}$ is such that $\{\tilde{Q}^i_t\}_{t\geq t'} = \{Q^i_t\}_{t\geq t'}$ for $\{M_t\} = \{\tilde{M}_t\}$ and $\{A_t\} = \{\tilde{A}_t\}$, as defined on Section II.D*

We can now define application strategies:

Definition 4. *An application strategy is a function $\alpha^i : \mathcal{H}_i \times \Pi_{j\neq i}\mathcal{H}_j \to (t_m, \infty]$, specifying the next application time.*

Note that each strategy α^i imply a function a^i_t representing the actions to adopt or not, therefore with codomain $\{0,1\}$. The strategy depends on beliefs and history, but the action can be left as an arbitrary decision. We say that the α^i will **induce** actions after histories.

II.F Myopic Adoption and Equilibrium

If agents adopt as soon as they consider it better than never taking it up, they take up the good myopically.

Definition 5. *A type v^n agent i adopts **myopically** if it does so at the first time t^M_i satisfying:*

$$\mu_{t^M_i} v^n - (1 - \mu_{t^M_i}) = 0$$

Note that a principal can never expect to do better than when all players adopt myopically, which leads to the $\{M^O_t\}$ adoption path. This is the case because no agent would adopt before it is myopically profitable to do so in equilibrium and delays by any positive mass of agents can only lead to further delays from others, as then μ_t will increase relatively more slowly.

Agents can also **strategically** choose the best moment to adopt, given an adoption path, and that no breakdowns happen. I will assume that, given a *deterministic* adoption path M_t, each agent i must optimally choose an adoption time. Ω^i_t gives the space of all possible a^i functions from t onwards. Note that given the continuum of agents, to condition behavior on deterministic adoption paths is without loss of generality, as any resulting adoption coming from mixed strategies is equivalent to non-symmetrical Nash equilibria.

Define by $\mathcal{T}^{\alpha^i}$ the set of application times determined by α^i. Define also by $\mathcal{T}^{\alpha^i}_t$ all the times strictly after t that are in $\mathcal{T}^{\alpha^i}$, and by $R_{t,w}$ the probability, assessed at time t, that up to time w the agent will have adopted the innovation, absent breakdowns up to that point.

$$R_{t,w} = \sum_{k \in \mathcal{T}_t^{\alpha^i}}^{w} \tilde{Q}_k^i$$

Define a value of waiting for agent i, at time t, for strategy α^i, by $V_t^{\alpha^i}$, for any path $\{M_t\} \neq M^{\emptyset}$ and *absence of breakdowns* up to $\bar{t}^{\alpha^i} \equiv \max \mathcal{T}^{\alpha^i}$ so that:

$$V_t^{\alpha^i} = \sum_{s \in \mathcal{T}_t^{\alpha^i}} e^{-r(s-t)} \tilde{Q}_s^i (1 - R_{t,s}) \left(\mu_t v^n - (1-\mu_t) e^{-\int_t^s \beta M_\tau d\tau} \right)$$

Define Ω_t to be the set of all possible functions α^i. The value for a v^n type agent to waiting at time t is given, then, by $V_t^n = \sup_{\alpha_t^i \in \Omega_t} V_t^{\alpha_t^i}$, and agents adopt as soon as this value is greatest.

If $q_N < 1 - \bar{M}$, the principal can ignore these agents with lower type v^N and reach her target. Assume, then, without loss of generality, that $\bar{M} > 1 - \min_{i \in I} q_i$, and therefore the principal wants to see agents of all types adopting. Like the agents, the principal is forward-looking and discounts future payoffs by the rate $r > 0$. I will henceforth denote the fundamental variables of the model $\xi = (\mu_0, \beta, r, v^{\mathcal{N}}, q^{\mathcal{N}}, \bar{M})$, which I will call an *economy*.

Agents hold common beliefs $\{\mu_t\}$ about the state of the world, and $\varphi_t^i($ beliefs about adoption and application pathss, as defined above.

Definition 6. *For any supply path $\{S_t\}$, an* ***equilibrium*** *is a set of strategy profiles $\alpha_t^{i,*}$, adoption path $\{M_t^*\}$ and application path $\{A_t^*\}$ such that, for every agent i:*

1. *For every $(h_i^{t_m}, h_{-i}^{t_m}) \in \mathcal{H}_i \times \Pi_{j \neq i} \mathcal{H}_j$, strategies $\alpha^{i,*} : \mathcal{H}_i \times \Pi_{j \neq i} \mathcal{H}_j \to (t_m, \infty]$ maximizes $V_t^{\alpha^i}$.*

2. *If $V_t^n > \mu_t v^n - (1 - \mu_t)$, then the $a_t^{i,*}$ action function induced by strategy $\alpha^{i,*}$ is such that $a_t^i = 0$ for any i with type v^n. Otherwise, $a_t^i = 1$.*

3. *$\{M_t^*\}$ is consistent with $\alpha^{i,*}$ for each i, so that induced $a_t^{i,*}$ is such that $M_t^* = \eta\{i \in I | a_\tau^{i,*} = o_\tau^i = 1 \text{ for some } \tau \leq t\}$.*

4. *$\{A_t^*\}$ is consistent with $\alpha^{i,*}$ for each i, so so that induced $a_t^{i,*}$ is such that $A_t^* = \eta\{i \in I | a_t^{i,*} = 1\}$.*

5. *$M_t^* \leq S_t$ for every $t \geq 0$.*

Note that the above is a Perfect Bayesian Equilibrium.

With that said, I will define a supply plan to be optimal in a set $\mathbb{S}$ of feasible plans if no other plan in the same set can induce a higher payoff for the principal.

Definition 7. *A supply plan $\{S_t\}$ is* ***optimal*** *among those in a set $\mathbb{S}$ if no other plan $\{S_t'\} \in \mathbb{S}$ induces an equilibrium with a higher payoff than her preferred one induced by $\{S_t\}$.*

II.G Simplification of Equilibrium Strategies and Beliefs

Given any supply plan and any equilibrium, there is another equilibrium with the same M_t, with these off-path beliefs and off-path strategies: suppose that $S_t > M_t + A_t$ for some period t. This would mean that an agent expects any application to be successful at that period. If it fails, though, I assume that the agent discontinuously change their belief to $S_t = M_t + A'_t$ and that she is part of a 0 mass of individuals who apply unsuccessfully. In other words, she thinks that a mass of applicants exhausted the available supply at that point in time. They also believe that $S_{t'} = M_{t'}$ for any time $t' > t$ that is strictly lower than a new batch of supply is released.

This implies we only need to consider deviations assuming on path equilibrium adoption path M_t.

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III Free supply

This section considers a setting where the principal is forced to be passive, serving the good whenever an agent applies for it. One way to think of it is that she commits to a plan $\{S_t^F\}$, with $S_t^F = \bar{S} > 1$ for all $t \geq 0$. In other words, the set of feasible supply plans $\mathbb{S}$ is a singleton with only $\{S_t^F\}$. We will see that informational free-riding leads to adoption happening convexily over some intervals. Finally, we discuss how this setting, without any impediments on adoption, can generate a S-shaped adoption curve for an increasing number of types with distribution approaching a continuous uniform $v \in \mathrm{U}[\underline{v}, \bar{v}]$.

With free supply, we have that the probability of receiving the good, conditional on applying at time t, denoted by Q_t, is always equal to 1. Therefore, the set of times in which an agent applies, $\mathcal{T}^{\alpha^i}$ is a singleton that we denote by t^{α^i}. Therefore the value of waiting for agent i of a type v^n, at time t, for strategy α^i is given, for any path $\{M_t\} \neq M^{\emptyset}$, by:

$$V_t^{\alpha^i} = e^{-r(t^{\alpha^i} - t)}\left(\mu_t v^n - (1-\mu_t) e^{-\int_t^{t^{\alpha^i}} \beta M_\tau d\tau}\right)$$

Formally, the value for a v^n type agent to waiting at time t is given, then, by $V_t^n = \sup_{\alpha_t^i \in \Omega_t} V_t^{\alpha_t^i}$, and agents adopt as soon as this value is greatest.

We will use the following notation to represent the preferences that agents have over take-up at different points in time for a given adoption path $\{M_t\}$: $T \succeq^{v^n|M_t} T'$. In other words, a type v^n agent weakly prefers taking up at time T compared to a time T' when the adoption path is $\{M_t\}$. We will omit the adoption path and use the notation $T \succeq^{v^n} T'$ whenever the adoption path is clear. If both $T \succeq^{v^n|M_t} T'$ and $T' \succeq^{v^n|M_t} T$ hold, we say that $T \sim^{v^n|M_t} T'$. In some of the coming arguments, it will also be useful to consider the agent's preferences over two different adoption paths. In particular, product availability will be used by the principal to choose between adoption paths that are only feasible if agents prefer to pick up at particular points in time. As this will lead to inferences regarding feasible adoption paths, we will use that to argue that some supply plans are optimal. We say that $T \succeq^{v^n|M_t,M'_t} T'$ if a type v^n agent weakly prefers taking up the good at time T with adoption path $\{M_t\}$ than doing the same at time T' with adoption path $\{M'_t\}$. If both $T \succeq^{v^n|M_t,M'_t} T'$ and $T' \succeq^{v^n|M_t,M'_t} T$ hold, we say that $T \sim^{v^n|M_t,M'_t} T'$.

Any equilibrium adoption path must satisfy the following property: adoption must occur weakly earlier for types that have higher good-state valuation.[4]

Definition 8. *We say that an economy $\xi = (\mu_0, \beta, r, v^{\mathcal{N}}, q^{\mathcal{N}})$ has the* ***quasi-single crossing property*** *if for any two agents i, i' with i a v-type and i' a v'-type, with $v > v'$, $a^i_t - a^{i'}_t \geq 0$ for every $t \in [0, \infty)$.*

Lemma 1. *Any economy $\xi = (\mu_0, \beta, r, v^{\mathcal{N}}, q^{\mathcal{N}})$ satisfies the quasi-single crossing property.*

Intuitively, any economy without this property would have a point t' in the future where it is strictly better to apply for some type v^n but not for some higher type, contradicting the payoff structure. But then, no take-up would happen at time t for the type v^n. The formal argument can be found in A.1.

We also have that there is only one equilibrium aggregate adoption path with free supply, for any economy ξ:

Proposition 1. *For $\{S_t\}$ with $S_t = \bar{S} > 1$ for every t, and any economy ξ with $v^1 > v^M_0$, there is a unique equilibrium adoption path, denoted by $\{M^F_t\}$.*

The proof can be found in Appendix A.2

III.A One Type

Suppose that agents can apply and get the good at any point in time. This section will consider the case with a single valuation type $v > 0$, so that $N = 1$. This setting is quite similar to the one considered by Frick and Ishii (2023), with the two significant differences being the normalization of the mass of potential adopters and the lack of stochastic opportunities to adopt. We will see first how adoption will happen convexly, and the stock of adopters is always increasing up to exhaustion of the mass of potential adopters.

The $\mu_{t^*} v - (1 - \mu_{t^*}) = 0$ condition on myopic take-up leads to adoption either happening for all types at $t = 0$ or never, depending on whether v is greater or lower than $v^M_0 \equiv (1 - \mu_0)/\mu_0$ which can be seen as a threshold value for when agents want to take-up myopically. If $v > v^M_0$, all apply as soon as possible; if $v < v^M_0$, no agent is willing to adopt. If no positive mass of agents adopts, though, beliefs stay the same as the prior, and the game is just the same over time. Therefore, we have that:

$$M^O_t = \begin{cases} 1 \ \forall t \text{ if } v \geq v^M_0 \\ 0 \ \forall t \text{ if } v < v^M_0 \end{cases}$$

[4] A similar property is also found by Laiho, Murto, and Salmi (2022) and Frick and Ishii (2023)

I denote by v_t^M the threshold value of the type that would adopt myopically at time t, whether one that is in V or not. From the definition of myopic adoption, we get that $v_t^M \equiv (1-\mu_t)/\mu_t$. It is easy to see that, for any adoption path $\{M_t\}$, $v_t^M = v_0^M e^{-\int_0^{t^*} \beta M_\tau^F d\tau}$.

What about strategic take-up? Note first that if $v < v_0^M$, no player wants to take up at $t=0$, beliefs do not change, and $M_t^F = 0$. This is also the case in the general model with many times, by the same logic, and therefore if the highest type is such that $v^1 < v_0^M$, then $M_t^O = M_t^F = 0$ always. One can also see that this is the case by noting that $0 \le M_t^F \le M_t^O$ always and the above description of M_t^O when $v < v_0^M$.

Take M_t^F as given and assume that $v \ge v_0^M$. One can take a first-order condition on the value of waiting at time t and expecting to pick up at time t^*, given by $V_{t,t^*}^n = e^{-r(t^*-t)}\left(\mu_t v - (1-\mu_t)e^{-\int_t^{t^*} \beta M_\tau d\tau}\right)$:

$$\frac{\partial V_{t,t^*}^n}{\partial t^*} = -re^{-r(t^*-t)}\left(\mu_t v - (1-\mu_t)e^{-\int_t^{t^*} \beta M_\tau^F d\tau}\right) + e^{-r(t^*-t)} \beta M_{t^*}(1-\mu_t)e^{-\int_t^{t^*} \beta M_\tau^F d\tau} = 0$$

Dividing both side by μ_t and using the definition of v_t^M above, one can check that:

$$v = v_0^M e^{-\int_0^{t^*} \beta M_\tau^F d\tau}\left(\frac{\beta}{r} M_{t^*}^F + 1\right)$$

As long as $v < v_0^M\left(\frac{\beta}{r}+1\right) \equiv v_0^F$. I denote the value v_t^F as an agent's minimum threshold payoff to be indifferent between take-up at t or one instant later. If $v \ge v_0^F$, all agents prefer to take up at $t=0$. They are sufficiently optimistic to adopt from the starting time of 0. Note then that the above equation uniquely defines M_t^F. It goes from a positive value up to 1, reached at a time T^F.

One crucial point in our later discussion is that M_t^F increases **convexly** when $v \in (v_0^M, v_0^F)$. This is a consequence of the fact that as μ_t increases over time, only the perspective of greater and greater gains can lead to agents being willing to wait. This is stated below and the formal argument can be found in A.3.

Proposition 2. *If $v > v_0^M$, there is a unique free-supply equilibrium adoption path $\{M_t^F\}_t$, which is i) strictly increasing and ii)convex up to $T^F \equiv \min\{t | M_t^F = 1\}$.*

This is shown in Figure 2.

We say that whenever there is an interval $[t_1, t_2]$ in which agents of the same type adopt the innovation, we have **partial adoption**.

Definition 9. *If there is an interval $[t_1, t_2]$ with $t_2 > t_1$ in which a positive mass of agents adopt the good. We say that there is partial adoption on that interval.*

III.B General Number of Types

With many types, we have the same free-riding incentives. In general, adoption will be characterized by periods of *partial adoption* together with some periods of no adoption. You can find a graphical

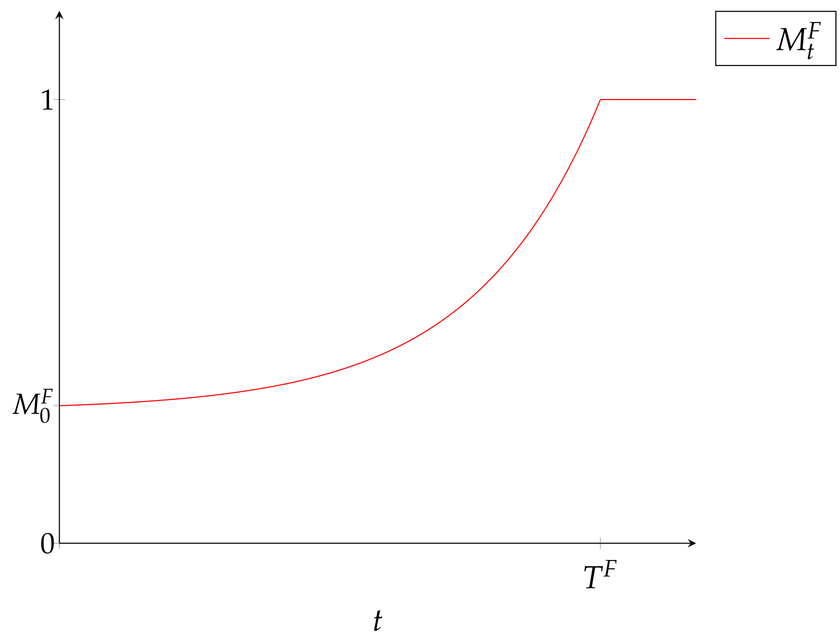


Figure 2: Free-Supply Adoption Path with Homogeneous Type

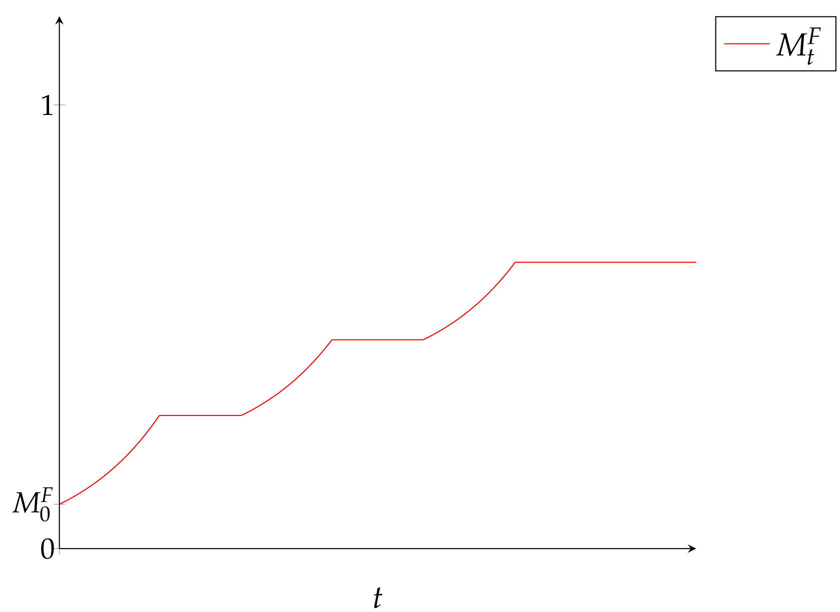


Figure 3: Free-Supply Adoption Path with Multiple Finite Types

representation in Figure 3. The following remark (proved on) summarizes the free supply take-up with a finite number of types:

Remark 1. *Any economy ξ can be partitioned in intervals over consecutive points t_1,t_2,t_3,t_4,...t_N in which M_t^F increases either convexly or stays constant.*

Adoption between two points (t_1,t_2) increases convexly whenever types free-ride on the information of their own type. As each type has only a q^n mass, no more agents apply right after t_2. The only point to discuss here is that these increasing intervals only have agents of the same type.

This comes from a straightforward strengthening of Lemma 8 that holds for free-supply only: higher types should apply to get the good strictly earlier. As the convex parts are all driven by indifference, only one type will apply to adopt at each of them.

III.C Convergence to Continuous Economy

In this section, we will focus on the case of economies with many types, converging to a continuum of types. I show first that if the distribution becomes arbitrarily close to one with a continuous distribution F, the free supply adoption path becomes arbitrarily close to the one for a continuum of types. I then analyze this continuum economy and see how it can generate an equilibrium adoption path with the famous S-shape, even without imposing restrictions on take-up availability. In particular, adoption has a **decreasing** second derivative. If a particular restriction on the prior is satisfied, take-up increases convexly at first and, towards the end, becomes concave.

Throughout this subsection, I will assume that $\bar{v} > v_0^M$, as if we were to have $\bar{v} < v_0^M$, no agent would be willing to adopt at time 0. If instead $\underline{v} > v_0^F$, all agents would want to adopt immediately.

I start by formally defining a sequence of economies converging to a continuous economy.

Definition 10. *A **continuous economy** ξ has a type set V such that v^n is distributed by a continuous cdf F with bounded support $[\underline{v},\bar{v}]$, $\underline{v} > 0$ and full support.*

Definition 11. *Take a sequence of economies $\{\xi^k\}$ differing only in valuation type sets (so that μ_0, β, r and $\bar{M}$ are kept constant for every k), $\{V^k\}$ with k types. We say that this sequence **converges** to a **continuous economy** if the distribution $\eta(v^n,q^n|v^n < x) = F^k(x)$ is such that $\lim_{k\to\infty} F^k(x) = F(x)$.*

One can note that what is being required above is that the sequence of economies must be such that the random variable denoting value types must converge in distribution to the continuous one.

Finally, I define convergence of adoption paths:

Definition 12. *Take a sequence of adoption paths $\{M^k\}$, each associated with an econnomy $\{\xi^k\}$. We say that it converges to an adoption path M^* if, for every $t \geq 0$ and $\epsilon > 0$, there is a K such that for any $k \geq K$, $|M_t^k - M_t^*| < \epsilon$.*

The first result shows that the adoption curve of an economy ξ^k with high k in such a sequence is arbitrarily close to the one for a limit continuous one (proof on Appendix A.4.

Proposition 3. *Take a sequence of economies $\{\xi^k\}$ converging to a continuous economy ξ^*. The corresponding sequence of free supply adoption paths $\{M^{F,k}\}$ converges to $\{M^{F,*}\}$, the unique free supply adoption path of the economy ξ^*.*

This result will be relevant in a later section, when we will consider an economy with many types, converging to a general continuos distribution F with bounded and full support.

I first discuss myopic adoption (proved in Appendix A.5):

Proposition 4. *When $v \sim U[\underline{v}, \bar{v}]$, the myopic adoption path is M_t^O is increasing and convex for $v_t^M \geq \bar{v}/2$ and concave for $v_t^M \leq \bar{v}/2$. Therefore, there is a threshold $\bar{\mu}_0$ such that if $\mu_0 \leq \bar{\mu}_0$, M_t^O initially increases convexly. If $\underline{v} < \bar{v}/2$, it will eventually become concave.*

Things work out similarly for the strategic adoption path:

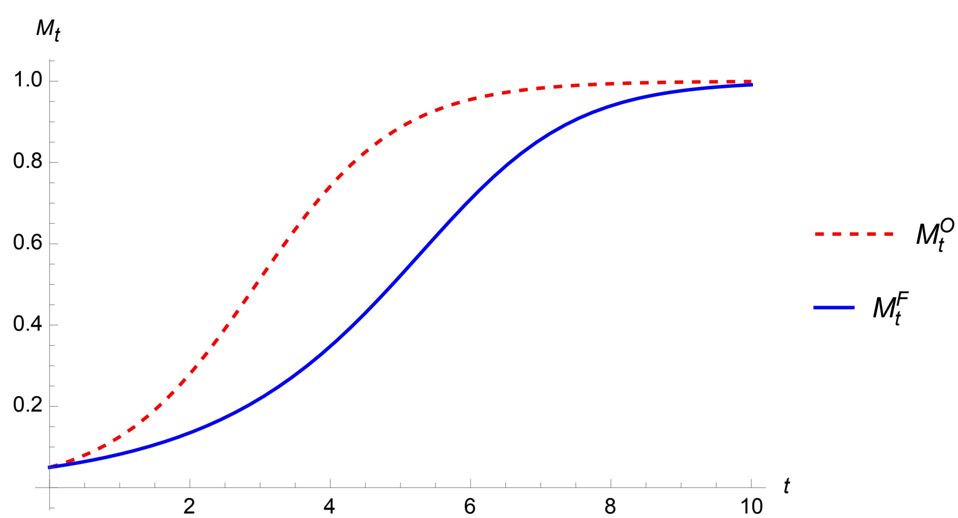
Proposition 5. *Take an economy ξ with $v \sim U[\underline{v}, \bar{v}]$ and. There is a threshold prior $\bar{\mu}_0$ such that if $\mu_0 < \bar{\mu}_0$ and $\bar{v} - \underline{v} > v_0^M$, the unique free-supply equilibrium adoption path M_t^F increases convexly up to some t_I and concavely after that.*

The formal proof can be found in Appendix A.6.

As an uniform distribution of types is both concave or convex, this illustrates that strategic considerations and heterogeneity are enough for the S-shaped adoption curve, that, as mentioned, is ubiquotous in adoption studies.

The following figure plots the adoption myopic and equilibrium adoption paths,M_t^O (green) and M_t^F (blue), for particular values of the parameters $\mu_0, \beta, r, \underline{v}, \bar{v}$[5]. As the remark shows states, the decreasing second derivative is a general trend of adoption curves for a uniform distribution. For these values, though, we see that we have myopic concave adoption together S-shaped strategic adoption.

Figure 4: S-shaped adoption curves for uniformly distributed valuations



As can be noted, for these parameters, we have a S-shaped equilibrium adoption curve.

[5] Note that these five parameters fully characterize the economy for a uniform distribution of types

IV Supply-Restricted Outcomes

In this section, I will present the main results of the paper, which show how a principal can use supply restrictions to reach an adoption target earlier. I first characterize how scarcity may lead to first-best for the principal whenever there is only one type of agent. In particular, arbitrarily low levels of scarcity lead to the unraveling of all free-riding equilibria with partial adoptionI next show that optimal supply plans exist and some are simple, with at most as many batches as types. I then go on to discuss how supply restrictions can always benefit the principal. Finally, a characterization of optimal supply plans for two, or three types is presented.

IV.A Optimal Supply with One Type

For this subsection, we will assume that the set of valuations V is a singleton with a unique element represented by v. This setting will clarify how powerful supply restrictions can be.

Suppose that the principal was to set $S_0 = \bar{M} < 1$. What would be the induced equilibrium take up path $\{M_t^E\}$?

First, note that we must have a positive mass of applicants in finite time so that $M_t > 0$ for some $t < \infty$. Otherwise, any agent would want to apply and get the good at time $t = 0$.

Suppose now that $t^* = \inf\{t | M_t^E > 0\} > 0$. Then, any agent i that applies at t^* has the same belief $\mu_{t^*} = \mu_0$ but, given discounting, a lower payoff. Therefore, any agent would rather deviate and apply at $t = 0$ instead.

The argument above establishes that any equilibrium take-up path must be such that $M_0^E > 0$. This means that we must have $T^E = \inf\{t | M_t^E = \bar{M}\} < \infty$ so that all will eventually get the good.

Consider now the mass of applicants at T^E. If $A_{T^E} > S_{T^E} - M_{T^E}^-$, a proportion $(S_{T^E} - M_{T^E}^-)/A_{T^E} < 1$ of the mass of applicants is randomly selected to receive the good. But then any such applying agent would prefer instead to apply and get the good $\epsilon > 0$ earlier. If, on the other hand, we have smooth take-up up to T^E, given that $\bar{M} < 1$, there is an agent i who ends up without the good but could apply at T^E and be better off instead. Therefore, we cannot have $T^E > 0$ and all apply to get the good at $t = 0$. The following theorem, proved on A.7, states this result:

Theorem 1. *For any economy ξ with $\bar{M} < 1$, $N = 1$ and $v > v_0^M$, $T^E(\bar{M}) = 0$*

This means that *arbitrary* amounts of scarcity can lead to myopic take-up behavior. Whenever supply is exactly equal to the mass q^n of v^n-type agents, we have two equilibria: one with informational free-riding and one without. The result above tells us that the former is unstable, and we are justified to focus on the latter instead.

IV.B Multiple Equilibria

As shown in the previous section, *arbitrary* levels of scarcity lead to a unique equilibrium without free-riding done by agents of one type on the adoption of agents of that same type. This creates the following discontinuity: for any scarcity level $\epsilon > 0$, we have immediate take-up from that type, but when ϵ reaches precisely 0, we have exactly two equilibria: 1) a symmetric one in which all agents of the same type adopt at the same time and 2) one in which agents free ride. The first one is still an equilibrium by the assumption that if adoption reaches the supply target, no agents can apply to adopt as long as no new mass is made available.

Similarly, if agents are indifferent between adopting at a point in time or never, we might have multiple equilibria because the mass that adopts immediately might be different. This is a very unstable equilibrium, though, for similar reasons as the one laid out in the last paragraph. In particular, a plan laid out one instant dt later would create incentives for all such agents to apply immediately. I also rule out this multiplicity by focusing on equilibria in which, for that case, agents of the same type choose the same action.

Whenever a plan induces multiple equilibria with different strategies for players of the same type in one of them, I assume that the principal is able to induce her preferred one.

Remark 2. *Suppose that a plan $\{S_t\}$ induces multiple equilibria. We assume that the principal is able to induce her preferred one as the outcome.*

IV.C Simple Plans

The discussion above suggests that creating batches for types to adopt immediately maximizes the benefits of scarcity. By doing this for a finite set of types, we have what we call **simple plans**. I formalize this notion below:

Definition 13. *A supply plan S_t is defined to be **simple** if, for N types, it has 1) up to N points with positive supply mass, and 2) it induces an equilibrium adoption path in which take-up only happens at this points in time in time in which a batch is released.*

It is the case that 1)simple supply plans always exist (and therefore an equilibrium always exists), and 2)to focus on simple supply plans is without loss of generality:

Proposition 6. *There always exist simple supply plans. Moreover, it is without loss of generality for the principal to focus on simple supply plans.*

The intuition for the result comes from the fact that there cannot be three different periods in which agents of the same type adopt the innovation. The complete proof can be found on A.8.

With the results from this and the previous section, we can discuss new insights from adding more types.

IV.D Supply Restrictions Always Benefit the Principal

No matter the distribution of types, it is the case that a simple supply plan imposing some restriction on supply, enough to reach the adoption target, is strictly beneficial for the principal.

Proposition 7. *For any economy ξ, there is an equilibrium with supply restriction that is strictly preferred by the principal to the free-supply equilibrium adoption path.*

The argument goes by construction: a supply plan with $S_t = \bar{M} < 1$ for any $t \geq 0$ will lead to a strictly greater payoff for the principal. The reason is analogous to that from Theorem 1: arbitrary restriction of supply for the last type that must be persuaded to adopt will lead to immediate adoption for that type, benefiting the principal. The full proof is in Appendix A.9

IV.E Optimal Supply with Two Types

With heterogeneous types, earlier take-up from higher types leads to earlier take-up from lower types. This section will characterize optimal supply plans for two types $v^1 > v^2$, illustrating these added benefits.

We start by reminding that we can focus on simple plans, with immediate take-up of any batches of supply and up to 2 batches, by Proposition 6.

We can rule out uninteresting trivial cases:

1. As noted in the previous section, if $v^1 < v_0^M$ the only equilibrium take-up path is $M_t^E = 0$ for any $t \geq 0$.

2. If $v_2 > v_0^M$, both types of agents are willing to adopt at time 0 myopically, and the principal can use arbitrary scarcity to have all apply at $t = 0$, as in the one-type case.

Suppose instead that $v^1 > v_0^M > v^2$. This means high-types are willing to take up the good myopically, and the low-types are not. Remember that we assume that $\bar{M} > 1 - \min_{i \in I}$ so the principal does not want to ignore a type to reach her target adoption rate.

Define by $T_1^D(m_1)$ the set of times a v^1 type is indifferent between adopting at this time with probability $(\bar{M} - m_1)/(1 - m_1)$ after a mass m_1 of agents adopts at time 0 and no agents adopt between 0 and $T_1^D(m_1)$. This probability comes from the fact that at this time all type v^2 agents and those who did not apply would apply to get the good, and therefore we would see rationing. $T_1^D(m_1) \neq \emptyset$ for any m_1 and is always finite. Define also $T_2^M(m_1)$ to be the time in which any v^2 types is willing to adopt myopically when a mass m_1 of agents adopting at time 0 and no agents adopting between 0 and $T_2^M(m_1)$. Note that $T_2^M(m_1)$ is a singleton.

Suppose first that the highest element of $T_1^D(q_1)$, denoted by $\bar{T}_1^D(q_1)$, is strictly lower than $T_2^M(q_1)$. Then, the principal can set $S_0 = q_1$. By the argument above, all v^1-type agents will apply at $t = 0$. But if $S_{T_2^M(q_1)} = q_1 + q_2$, all v^2-type agents will apply to get the good myopically at time $T_2^M(q_1)$. This plan induces myopic take-up (remember Remark 2), making it optimal for the principal among all possible supply plans.

Suppose now instead that $\bar{T}_1^D(q_1) > T_2^M(q_1)$. The planner is no longer able to reach the adoption target at time $T_2^M(q_1)$, as then any v^1 type would rather apply to get the good at this time instead of $T_2^M(q_1)$. Note though that $\bar{T}_1^D(m_1)$ is *continuous* in m_1 (more learning is available creates incentives to delay take-up longer), as well as $T_2^M(m_1)$. It is easy to see that the latter is always decreasing in m_1, as less learning has happened to induce the lower types to take up myopically. The behavior of $\bar{T}_1^D(m_1)$ is more ambiguous, though: an increase in m_1 has a positive effect on it by increasing the informational free-riding opportunities, but a negative one by its negative effect on $(\bar{M} - m_1)/(1 - m_1)$, the probability of getting the good. As, by assumption, $\bar{T}_1^D(q_1) > T_2^M(q_1)$ and for $m_1 \to 0$, we have that $\bar{T}_1^D(m_1) \to 0$ and $T_2^M(q_1) \to \infty$, we know that the set of points for which $T_1^D(q_1)$ and $T_2^M(q_1)$ intersect is non-empty. We can take the lowest m_1^* in which an element of $T_1^D(m_1^*)$ and $T_2^M(m_1^*)$ intersect. The principal can then set $S_0 = m_1^*$ and then serve the remainder at $T_2^M(m_1^*)$.

This plan is optimal: any optimal plan must have v^2 types taking up myopically. Given that, we need to have v^1 types weakly preferring to take up at $t = 0$ then with the second batch.The following theorem summarizes the discussion in the previous paragraph (proved in A.10):

Theorem 2. *The optimal supply plan when $N = 2$ and $v^1 > v_0^M > v^2$, consists of using two batches at times* 0 *and t_2, with sizes $m_2 = \bar{M} - m_1$ and $m_1 = q_1$ if $\bar{T}_1^D(q_1) < T_2^M(q_1)$, or $m_1 = m_1^*$ for the lowest point in $T_1^D(m_1)$ such that $T_1^D(m_1^*) \cap T_2^M(m_1^*) \neq \emptyset$.*

You can see a graphical representation of both cases in Figure 4, where we also compare outcomes to the free-supply one M_t^F.

IV.F Optimal Supply with Three Types

The addition of a third type gives us new insights. In particular, it clarifies that the condition for not serving all highest types at time zero will still holds when there are types other than the highest and the lowest. We will also be able to see that even without that, in some situation it is better to leave the mid-type taking up together with the lowest one. Finally, we will learn that whenever we exhaust the free-riding opportunities for the high-type, there is a trade-off between getting earlier or larger adoption of the mid-types, and that this tension will usually benefit the latter.

Suppose that there are three possible types $v^1 > v^2 > v^3$. I consider, for simplicity, the case when $\bar{M} = 1$, the same as in the last section. As before, if $v^1 < v_0^M$, no agent would ever adopt, and if $v^3 > v_0^M$, the principal can simply use an arbitrary amount of scarcity to reach her target at time 0. This leaves us with the following cases:

1. $v^1 > v_0^M > v^2 > v^3$

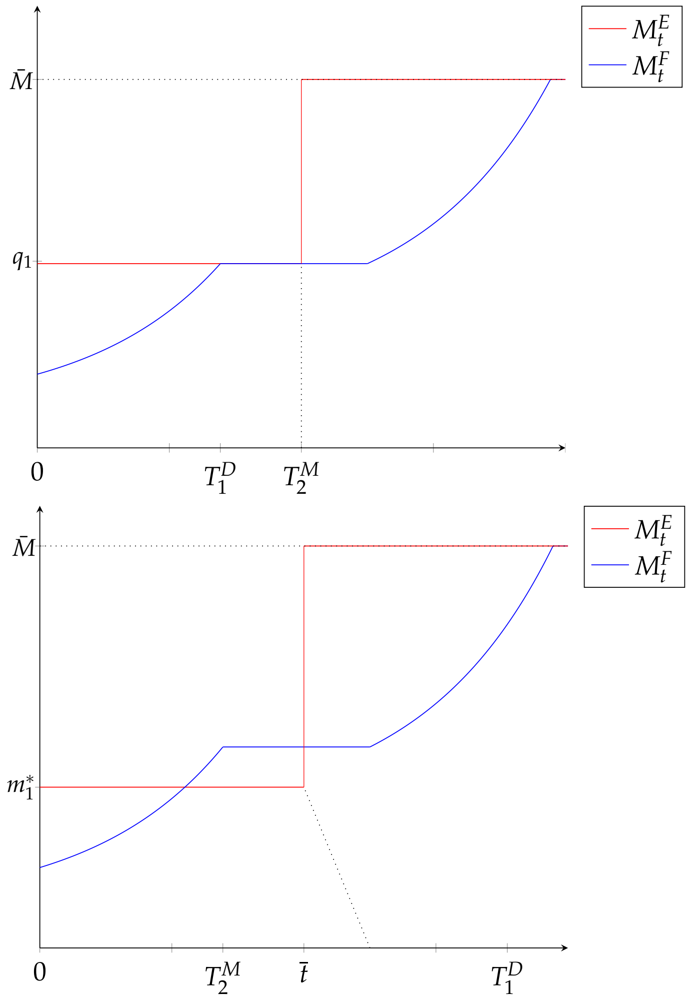


Figure 4: Two-Types Case

2. $v^1 > v^2 > v_0^M > v^3$

We will break down the optimal supply algorithm for the first case. It is not without loss of generality to do so, but the steps for the second one are very much analogous.

We start by reminding that to consider only simple supply plans is without loss of generality. In this case, this means that at most three batches, inducing immediate adoption, are enough.

For the preference relation below, we omit the adoption path considered, as they all refer to the equilibrium adoption path induced by the supply plan so that $T \succeq^{v^n} T'$ means that $T \succeq^{v^n|M_t^E} T'$. Before going through the algorithm itself, let's first define some important variables:

- $T_1^D(m_1) \neq \emptyset$ is the set of times in which v^1 is indifferent between adopting at time 0 for sure and this time (potentially competing with some lower valued agents) when a mass m_1 adopts at time 0. So $0 \sim^{v^1} t$ for any $t \in T_1^D(m_1)$.

- $T_2^E(m_1)$ is the preferred time for a v^2-type agent to adopt up given that a m_1 mass has done so at $t=0$ and no new mass has done it after that.
- $T_3^M(m_1,t_2,m_2)$ is the type in which the v^3 type is indifferent between adopting now or never again, given that a mass m_1 decided to do so at time 0 and an extra mass m_2 at time t_2.

We can now go to the full three-type algorithm for a limiting economy:

1. Compare $\bar{T}_1^D(q_1) \equiv \max\{t \in T_1^D(q_1)\}$ and $T_3^M(q_1,0,0)$. If the former is greater than the latter, release, at time 0, a batch of size m_1^* in which the v^1-types are indifferent. Otherwise, go to next step.
2. Check if $T_2^E(q_1) > T_3^M(q_1,0,0)$. If so, release batches at times 0 and $T_3^M(q_1,0,0)$, with $m_1=q_1$ and $m_2=1-q_1$. Otherwise, go to next step.
3. Check if $\bar{T}_1^D(q_1) \prec^{v^2} T_3^M(q_1,0,0)$. If so, no batching for the v^2 types is profitable. Otherwise, go to next step.
4. If $T_3^M(q_1,\bar{T}_1^D(q_1),q_2) \succ^{v^2} \bar{T}_1^D(q_1) \succ^{v^2} T_3^M(q_1,0,0)$, there are two options: If $T_2^E(q_1) > \bar{T}_1^D(q_1)$, go for the former. Otherwise, go for m_2^* at $\bar{T}_1^D(m_2^*)$ making v^2 indifferent.
5. If $\bar{T}_1^D(q_1) \succ^{v^2} T_3^M(q_1,\bar{T}_1^D(q_1),q_2)$, then pick the lowest m_1^* such that $T_1^D(m_1^*) \sim^{v^2} T_M^3(m_1^*,T_1^D(m_1^*),q_2)$. Compare this to $T_2^E(q_1)$. One of these two is optimal.

I briefly discuss the intuitive argument for each point below:

1. Suppose that adding a mid-batch is better for the principal. Then, by definition, it will lead to a higher payoff to types v^1 than before, as it implies the same amount of learning happening earlier. But then no v^1 type would want to take up at time 0 and therefore we cannot have this adoption path as induced in equilibrium.
2. If the condition is met, then $T_3^M(q_1,\bar{T}_1^D(q_1),q_2) \succ^{v^2} \bar{T}_1^D(q_1) \succ^{v^2} T_3^M(q_1,\bar{T}_1^D(q_1),0)$. Adding a new batch with positive mass take-up would lead to an even more relatively desirable $T_3^M(q_1,\bar{T}_1^D(q_1),q_2)$, though—contradiction with take-up at the new batch.
3. Suppose that any batch for v^2 can make things better for the principal. This means the final batch will come even earlier, with the same amount of learning. But then v^2 must strictly prefer taking up at this last batch, a contradiction.
4. A second batch with q_2 at $\bar{T}_1^D(q_1)$ is not feasible (they would all prefer to take up with v^3). Bringing more v^2 types later rather than sooner is always better. However, one should never decrease m_1 to get more v^2 earlier.

5. This comes from the concavity of the objective function being minimized between these two points and the fact that the minimum cannot be after $T_2^E(q_1)$ or reversing the preference relation.

I note first that the restriction on three batches is not vacuous, and two batches are not enough in some cases, as in the example in Appendix B.1. I name the algorithm above the **three-type optimal algorithm** (TTOA) and the supply plan induced by it $\{S^{TTOA}\}$. With these two concepts, I state the theorem for this section (proved in Appendix A.11):

Theorem 3. *If $v^{\mathcal{N}} = (v^1, v^2, v^3)$ and $v^1 > v_0^M > v^2 > v^3$, then $\{S^{TTOA}\}$ is optimal.*

IV.G Increasing Number of Types

Although relevant, the new insights coming from the two and three-type models are not enough to give us a sense of what characteristics the optimal supply plan would have with an unbounded number of types. I will give insight into this question by using the construction on subsection III.C

Proposition 6 shows that for any finite k number of types, there is an optimal plan with up to k batches. I will now show that the ratio of batches to types converges to a number strictly lower than 1, whenever a sequence of economies $\{\xi^k\}$ converges to a continuous economy with valuation distribution F having bounded and full support:

Proposition 8. *Take a sequence of economies $\{\xi^k\}$ converging to a continuous economy ξ^* with associated distribution of values F bounded with full support. There is a $\gamma < 1$ such that for any $\epsilon > 0$, there is a K such that for every $k \geq K$, there is an optimal supply plan such that the ratio of batches to types is lower then $\gamma + \epsilon$.*

This result builds on Proposition 3, and remarks that for a continuous economy with full support, a fraction would want to adopt myopically together with the lowest type $\underline{v}$. The full proof can be found in Appendix A.12.

V Discussion and Extensions

V.A Discussion

The results from the previous section give us insights into how a planner can achieve a target adoption rate as soon as possible. As discussed, this objective is common for diverse problems, such as reaching immunization rates, dealing with pests, or getting a critical mass of adopters for a matching platform. Given the distribution of valuations, the principal knows when to add new batches of the good and how many units to use.

As discussed in the literature review, there are other reasons why a principal may want to restrict supply (notably for persuasion purposes). Still, the question might add to the arsenal of tools available for public policymakers dealing with informational free-riding.

There is a clear gap between the interests of the principal and that of agents in the model. In particular, the former wants take-up to happen no matter what, and a social planner would rather have no adoption if $\omega = b$.

In the model, welfare does not increase with faster-take-up: players take up myopically and might be more rapidly adopting a bad product.

Remark 3. *Take a finite economy ξ. The optimal supply plan is:*

1. *Welfare **neutral** for all types $v > v_0^E$.*
2. *Welfare **decreasing** for the lowest type.*
3. ***Ambiguous** for all other types.*

Intuitively, the highest types get the same payoff from adopting in both cases. The lowest type was able to free-ride and achieve a positive payoff in the free supply equilibrium. However, the principal extracts that surplus when scarcity makes immediate myopic adoption the unique equilibrium for these agents.

For all other types, results are ambiguous. This is the case due to the combination of two effects: 1) earlier take-up from higher types benefits any mid-type; 2) the supply restriction for these types hurts their outcomes. With three types and an optimal supply plan with two batches, it is easy to see how the second source can dominate. If there are three batches and the principal wants to "please" the mid-types, on the other hand, one can see that the first may dominate. The entire examples of each case can be found in Appendix B.2.

One can also consider the case of a social planner with the same tools as the principal. It is the case that with only one type, a social planner would not be able to make Pareto improvements. This is clearly the case for any highest type. With two types, though, if, for example, $T_1^D(q_1) < T_2^E(q_1)$, the v^1 types are indifferent when compared to free supply, but the v^2 types are strictly better off, as they can learn more quickly. With three types, as long as $T_1^D(q_1) < T_2^E(q_1)$, there are welfare gains for the mid-type. By batching at the point $T_2^E(q_1)$ and not batching for the lowest types, it is the case also that there are gains for type v^3. Note that this is similar to the optimal supply plan with three types whenever some mid-types want to adopt in a second batch, but not all. There are welfare gains for mid-types in that case as well, but the principal "extracts" the benefits for the lowest type by making them adopt too early.

Note also that if the objective of the planner is not to reach a target $\bar{M}$ as soon as possible but rather to get a preferred adoption path M_t, with strict preferences for paths that are weakly above at each point in time, then, by definition, scarcity, by either supplying fewer units or later, can never be desirable. Note that the myopic take-up path M_t^O is strictly preferred to any strategic one M_t^E. Define an *optimal supply path* as one that is not strictly preferred by any other. The hands-free supply path is optimal, then.

V.B Extensions

In this section, I discuss a few extensions of particular interest. In particular, cases in which supply can be contingent on adoption history, an extension in which there are different regions with different arrival rates of news, as well as the case of two products that are ex-ante equal. I will argue that adoption-contingent would not change the results much, and the principal can do no better than using time-contingent supply plans. With two regions, despite the preference for take-up from the one for which the arrival rate of news is larger, the principal must reach its target simultaneously in both regions. Finally, I show that there are situations in which the principal cannot benefit from a second good, even though, in principle, it could be used to hedge against the possibility that one of them is shown to be of low quality.

V.B.1 Take-Up Contingent Supply Plans

Suppose that the principal, instead of committing to time-dependent only supply plans, is able to commit to adoption-dependent ones. One could imagine that this would be a very powerful tool for the principal, as threats of not providing the good if the rate of adoption is low could have big effects on incentives to adopt early.

I will argue, though, that the principal would not benefit from the ability to commit to these types of plans. The reasoning goes as follows: an effective threat for slow adoption would require a certain level of supply being released at a point in time, followed by new batches released in the future conditional on that adoption. As the principal is able to induce her preferred equilibrium with time-contingent only supply plans, though, she can promise the same supply plan with only time contingency.

Importantly, the argument above relies on the continuum assumption and the fact that types are not observable. Observable types could make adoption contingency significantly more powerful: to supply just the mass of agents for a certain type would lead to myopic adoption as an equilibrium.

V.B.2 Different Regions

I discuss briefly what would happen if two regions with different arrival rates of bad news ($\beta_1 > \beta_2$), but with such breakdown visible for both and the same payoff distributions and other parameters. The principal needs to have adoption from all types in both regions and can restrict supply in any particular one.

Despite the potential advantage of the region with the higher beta, the target adoption must be reached *simultaneously*. This is the case because the lowest types in both regions must adopt myopically, and they share both a prior and beliefs.

This suggests that areas with less access to health care (such as rural areas), and in which the assessment of side effects of a vaccine could be slower to get, should not be left to have skeptics taking later on if one needs to reach adoption targets fast, as is the case for any rapidly mutating viral disease, as COVID-19.

V.B.3 Multiple products

Suppose now that the principal has more than one product at her disposal, ex-ante equal. This means that the rate of arrival of bad news or the prior about both is the same for both products. I will analyze the case of two products in a populations with two types $v^1 > v_2$. The potential advantage of having two products is that the principal can release another if one is revealed to be bad. Therefore, if supply can be made conditional on realizations, it is always optimal to have two products.

However, this might not be the case when supply plans cannot be released with that type of contingent power. In particular, scarcity might be compromised whenever another product is to be released in the future.

Note first that the best use of two goods is to release them simultaneously and at the same amount. This is because hedging against the risk that one of them is shown to be bad is optimal only when they are equally treated.

Take an economy where the principal is already at its first-best with one product. This means, with two types, that $T_1^D(q_1) < T_2^M(q_1)$. Note that perfect hedging with two goods leads to take-up of v^2 types happening at time $2T_2^M(q_1)$. To see this, remember that v^2 types will adopt myopically so that $v_{\bar{t}}^M = v^2$. With half the learning, it takes exactly double the time for the low types to adopt, as $\bar{t} = (1/\beta q_1) ln(v_0^M / v^2)$. Therefore, even without considering the fact that supply restrictions can no longer be effective for the final time, there are no gains from using two products.

For an example of a setting in which the presence of two types leads to a gain for the principal, consider an economy in which only half the q_1 mass of v^1 types are served at time 0 in the optimal simple plan with one type of product only. To release $q_1/2$ of type b_1 and $q_1/2$ mass of type b_2 product at time 0 should lead to the benefits of hedging without any delay in take-up before any v^2 type does. For μ_0 low enough, this is preferable to the simple plan with one product. This illustrates a case in which the advantages of having two products are potentially greater.

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APPENDIX: FOR ONLINE PUBLICATION

Ricardo Fonseca

A Proofs

This document presents proofs omitted in the main text and additional theoretical results.

A.1 Lemma 1

Suppose that, in contrast to the statement of the lemma, we have an equilibrium adoption path in which two agents i,i', with the former being a type v^n, and the latter of type $v^{n'} < v^n$ applying to adopt the innovation strictly earlier.

This means that, for some times t,t', with $t<t'$, we have that:

$$V_t^{n'} \geq V_{t'}^{n'}$$

Which implies that:

$$e^{-rt}\left(\mu_t v^{n'} - (1-\mu_t)e^{-\int_0^t \beta M_\tau d\tau}\right) \geq e^{-rt'}\left(\mu_{t'} v^{n'} - (1-\mu_{t'})e^{-\int_0^{t'} \beta M_\tau d\tau}\right)$$

So that

$$v^{n'}\left(e^{-rt}(\mu_t v^{n'}) - e^{-rt'}(\mu_{t'})\right) \geq e^{-rt'}\left((1-\mu_t)e^{-\int_0^{t'} \beta M_\tau d\tau}\right) - e^{-rt}\left((1-\mu_t)e^{-\int_0^{t} \beta M_\tau d\tau}\right)$$

So that

$$v^{n'} \geq \frac{e^{-rt'}\left((1-\mu_t)e^{-\int_0^{t'} \beta M_\tau d\tau}\right) - e^{-rt}\left((1-\mu_t)e^{-\int_0^{t} \beta M_\tau d\tau}\right)}{\left(e^{-rt}(\mu_t v^{n'}) - e^{-rt'}(\mu_{t'})\right)}$$

But note that this means that, for $v^n < v^{n'}$, we have that:

$$v^{n} \geq \frac{e^{-rt'}\left((1-\mu_t)e^{-\int_0^{t'} \beta M_\tau d\tau}\right) - e^{-rt}\left((1-\mu_t)e^{-\int_0^{t} \beta M_\tau d\tau}\right)}{\left(e^{-rt}(\mu_t v^{n'}) - e^{-rt'}(\mu_{t'})\right)}$$

So that $V_t^n \geq V_{t'}^n$, which contradicts the fact that $a_t^i = 1$ and $a_{t'}^i = 0$.

It is easy to see that an analogous result holds for supply-restricted settings as long as they respect anonymity, as then Q_t, the probability of having a successful application does not depend on the type.

A.2 Proposition 1

We must first show that a free-supply equilibrium adoption path exists and is unique when $v^1 > v_0^M$.

If $v^N > v_0^E \equiv v_0^M\left(\frac{\beta}{r}+1\right)$, all types would prefer to adopt immediately and therefore $M_0^F = 1$ and $M_0^F = 1$ for every $t \geq 0$ is the unique equilibrium adoption path.

Otherwise, taking the FOC for the agent problem, we get that adoption happens for all types with valuations higher than v_0^F, defined by:

$$v_0^F = v_0^M\left(\frac{\beta}{r}M_0^F + 1\right)$$

Where M_0^F is the mass of applicants with valuation strictly higher than v_0^F plus potentially some amount of v_0^F participants to make it hold with equality, in case v_0^F is a type in V.

From the description above, it is easy to see that such v_0^F is uniquely defined. Note, though, that for those who have not adopted up to time t, we have:

$$v_t^F = v_t^M\left(\frac{\beta}{r}M_t^F + 1\right)$$

, for analogously defined M_t^F. As $v_t^M = (1-\mu_t)/\mu_t$ and $M_0^F > 0$, we know that $\mu_t \to 1$ and therefore v_t^F decreases to $\underline{v}$. Therefore, eventually, all types adopt the innovation. This show existence and uniqueness of $\{M_t^F\}$.

A.3 Proposition 2

The proof that the equilibrium adoption path $\{M_t^F\}$ for an economy ξ has the properties of existence and uniqueness comes directly from Proposition 1. That it is strictly increasing over time and convex for a homogeneous valuation economy follows directly from Theorem 1 from Frick and Ishii (2023). The only difference between the present setup and theirs is that we do not have stochastic adoption opportunities. However, for the indifference region, which is the entire one in our setting for $v > v_0^M$, the proof that the equilibrium path is convex does not depend on the random opportunities to adopt, so it holds analogously.

To see that it is strictly increasing, suppose the path is constant in some interval $[t_1, t_2]$ with $t_2 > t_1$ and $M_{t_2}^F < 1$. Then, it must be the case that for some individuals waiting at a time $t \in (t_1, t_2)$ is at least as good as adopting at time t_2. By homogeneity of the value of adoption in the good state, though, and the fact that $M_{t_1}^F > 0$, we must have $V_{t_1}^n \leq \mu_{t_1} v^n - (1-\mu_{t_1})$. But then, by Proposition 1, we must have $V_t^n < \mu_t v^n - (1-\mu_t)$, contradicting the assumption that waiting at t is preferred.

A.4 Proposition 3

The first consideration is to show that $\{M_t^{F,*}\}$, the free-supply adoption path for a continuous economy ξ^* exists and is unique.

To see that, suppose first that $\bar{v} \leq v_0^M$. Then $M^{\emptyset}$ is the unique equilibrium adoption path. Similarly, if $\underline{v} > v_0^F = v_0^M \left(\frac{\beta}{r}+1\right)$, all types would want to adopt immediately, and that would define the unique adoption path. Suppose, then, $v_0^F > \bar{v} > v_0^M$. Then, by full support, a positive mass is willing to adopt the good at time 0. This mass is defined by a type v_0^E just indifferent between adopting at time 0 and one instant later:

$$v_0^F = v_0^M \left(\frac{\beta}{r}(1 - F(v_0^F)) + 1 \right)$$

It is easy to see that such v_0^F must exist and is unique for $\bar{v} > v_0^M$. Note that $M_0^F > 0$. Analogously, for $t > 0$, we have:

$$v_t^F = v_t^M \left(\frac{\beta}{r}(1 - F(v_t^F)) + 1 \right)$$

This is also uniquely defined, and, as $M_0^F > 0$, $\mu_t \to 1$ and therefore $v_t^M \to \underline{v}$. Therefore, we have the existence and uniqueness of $\{M_t^{F,*}\}$.

For a sequence of economies $\{\xi^k\}$ converging to a continuous economy ξ^* with a distribution of valuation F, we must show that, for any t and $\epsilon > 0$, there is a K such that for any $k \geq K$, $|M_t^{F,k} - M_t^*| < \epsilon$, where $\{M_t^{F,*}\}$ is an equilibrium free-supply adoption path for the continuum economy.

Take $\epsilon > 0$, and k . Adoption at this time 0 implies that some type v^n had 0 as the maximizing point of:

$$V_0^n = e^{-rt'}(\mu_0 v^n - (1-\mu_0) e^{-\int_0^{t'} \beta M_\tau d\tau})$$

Note that for k great enough, we have that $M_0^{F,k} \sim M_0^*$. This implies that for $T > 0$, one can find a k large enough such that $M_T^{F,k} \sim M_T^*$, as the learning to happen up to T will be arbitrarily close to the one on the continuous economy, given that $\int_0^{t'} \beta M_\tau d\tau$ is clearly continuous on the adoption path $\{M_t\}$.

A.5 Proposition 4

Myopic behavior is governed by the following:

$$\dot{v}_t^M = -\beta \frac{\bar{v} - v_t^M}{\bar{v} - \underline{v}} v_t^M$$

With $v_0^M = \frac{1-\mu_0}{\mu_0}$ and $v_t^M = \frac{1-\mu_t}{\mu_t}$. To see why that is the case, Note that $v_t^M = v_0^M e^{-\beta \int_0^t M_\tau^F d\tau}$ for every t. Differentiating both sides with respect to t leads to $\dot{v}_t^M = -\beta v_t^M v_t^M$, which implies the equation above.

One can solve the differential equation and find that the solution is of the form:

$$v_t^M = \frac{k_2 \bar{v}}{k_2 + e^{t\bar{v}k_1}}$$

where k_1 and k_2 are constants:

$$k_1 = \frac{\beta}{\bar{v} - \underline{v}}$$

And k_2 is given by the initial condition:

$$k_2 = \frac{1-\mu_0}{\bar{v}\mu_0 - (1-\mu_0)} \tag{A.1}$$

From this equation, one can note the following:

1. v_t^M is decreasing
2. If $v_t^M \geq \bar{v}/2$, v_t^M is concave decreasing, and if $v_t^M \leq \bar{v}/2$, v_t^M is convex decreasing. To see that, note that

   $$\dot{v}_t^M = -k_1(\bar{v} - v_t^M)v_t^M$$

   And therefore:

   $$\ddot{v}_t^M = -k_1\dot{v}_t^M(\bar{v} - 2v_t^M)$$

   As $k_1\dot{v}_t^M = -k_1^2(\bar{v} - v_t^M)v_t^M < 0$, we have the result.
3. The inequalities from the previous point imply the opposite for the M_t^O: if v_0^M is high enough, it starts convex increasing, but eventually it becomes concave increasing.
4. The inflection point of M_t^O is given by t^I such that $v_{t^I}^M = \bar{v}/2$. Note that it might not be reached if v_0^M starts below $\bar{v}/2$ (people already start too optimistic), or if $\underline{v} > \bar{v}/2$, in which case take-up always happens convexly.

We conclude that $\{M_t^O\}$, the myopic adoption path, has an S-shaped adoption form for optimistic enough agents.

A.6 Proposition 5

If take-up happens in equilibrium, we must have the following:

$$v_t^F = \frac{1-\mu_t}{\mu_t}\left(\frac{\beta}{r}M_t^F + 1\right)$$

For the uniform example, this implies:

$$v_t^F = \frac{1-\mu_t}{\mu_t}\left(\frac{\beta}{r}M_t^F + 1\right)$$

This leads to:

$$v_t^F = \frac{1-\mu_0}{\mu_0} e^{-\beta \int_0^t M_\tau^F d\tau} \left(\frac{\beta}{r} M_t^F + 1 \right)$$

Using that $v_0^M = (1-\mu_0)/\mu_0$, we have:

$$v_t^F = v_0^M e^{-\beta \int_0^t M_\tau^F d\tau} \left(\frac{\beta}{r} M_t^F + 1 \right)$$

Differentiating both sides with respect to t, we get:

$$\dot{v}_t^F = -v_0^M \beta M_t^F e^{-\beta \int_0^t M_\tau^F d\tau} \left(\frac{\beta}{r} M_t^F + 1 \right) + v_0^M e^{-\beta \int_0^t M_\tau^F d\tau} \frac{\beta}{r} \dot{M}_t^F$$

So that:

$$\dot{v}_t^F = -\beta M_t^F v_t^F + v_0^M e^{-\beta \int_0^t M_\tau^F d\tau} \frac{\beta}{r} \dot{M}_t^F$$

As $M_t^F = \frac{\bar{v} - v_t^F}{\bar{v} - \underline{v}}$, we have that:

$$\dot{M}_t^F = -\frac{\dot{v}_t^F}{\bar{v} - \underline{v}}$$

Therefore:

$$\dot{v}_t^F = -\beta M_t^F v_t^F - v_0^M e^{-\beta \int_0^t M_\tau^F d\tau} \frac{\beta}{r} \frac{\dot{v}_t^F}{\bar{v} - \underline{v}}$$

Denote

$$k_1 = \frac{\beta}{r(\bar{v} - \underline{v})}$$

and

$$k_2 = \frac{\beta}{(\bar{v} - \underline{v})}$$

Which leads to:

$$\dot{v}_t^F = -k_2 (\bar{v} - v_t^F) v_t^F - k_1 v_0^M e^{-\beta \int_0^t M_\tau^F d\tau} \dot{v}_t^F$$

We conclude that:

$$\dot{v}_t^F = -\frac{k_2 (\bar{v} - v_t^F) v_t^F}{1 + k_1 v_0^M e^{-\beta \int_0^t M_\tau^F d\tau}}$$

Differentiating both sides with respect to t, we get:

$$\ddot{v}_t^F = -\frac{k_2(\bar{v}\dot{v}_t^F - 2v_t^F\dot{v}_t^F)(1+k_1 v_0^M e^{-\beta\int_0^t M_\tau^F d\tau}) - k_1 v_0^M \beta M_t^F e^{-\beta\int_0^t M_\tau^F d\tau}(k_2(\bar{v}-v_t^F)v_t^F)}{(1+k_1 v_0^M e^{-\beta\int_0^t M_\tau^F d\tau})^2}$$

We are interested in the sign on the right-hand side, which is equal to the sign of

$$-(\bar{v}\dot{v}_t^F - 2v_t^F\dot{v}_t^F)(1+k_1 v_0^M e^{-\beta\int_0^t M_\tau^F d\tau}) + k_1 v_0^M \beta M_t^F e^{-\beta\int_0^t M_\tau^F d\tau}(\bar{v}-v_t^F)v_t^F$$

Using the formula for $\dot{v}_t^F$, we get that this is equal to:

$$(\bar{v}-2v_t^F)k_2(\bar{v}-v_t^F)v_t^F + k_1 v_0^M \beta M_t^F e^{-\beta\int_0^t M_\tau^F d\tau}(\bar{v}-v_t^F)v_t^F$$

Which has the same sign as:

$$(\bar{v}-2v_t^F)k_2 + k_1 v_0^M \beta M_t^F e^{-\beta\int_0^t M_\tau^F d\tau}$$

Therefore we can focus on:

$$(\bar{v}-2v_t^F) + v_0^M \frac{\beta}{r} M_t^F e^{-\beta\int_0^t M_\tau^F d\tau}$$

Which is equal to :

$$(\bar{v}-2v_t^F) + (v_t^F - v_t^M) = \bar{v} - v_t^F - v_t^M$$

Clearly, if v_m^0 is large enough (and therefore μ_0 is low enough), the above expression is negative. As $\ddot{M}_t^F = -\ddot{v}_t^F/(\bar{v}-\underline{v})$ we have that adoption starts increasing convexly. The condition that $\bar{v}-\underline{v} > v_0^M$ guarantees that it will eventually become negative, as v_t^M will eventually reach $\underline{v}$. One can also note that in this case, there is a unique inflection point t_I in which $\bar{v} = v_{tI}^F + v_{t_I}^M$.

A.7 Theorem 1

We need only to show that $V_0 > \mu_0 v - (1-\mu_0)$. To see that, note first that $M^\emptyset$ is not an equilibrium, by the assumption that $v > v_0^M$. Therefore $\lim_t \mu_t = 1$ and there is a time T in which all agents strictly prefer to adopt $a_T = 1$. Take T to be the moment any agent last gets to take up the good. There are two options as to what happens at T if it is strictly greater than 0:

(i) $\eta(i|a_T^i = 1) > S_T$.

As described, a lottery will happen at T, and a fraction $Q_T \in (0,1)$ of agents will receive the good. But then there is an $\epsilon > 0$, so it is better for these players to apply at $T-\epsilon$. To see that, not that the payoff at T is given by:

$$\tilde{Q}_T e^{-rT}(\mu_T v-(1-\mu_T))e^{-\int_0^T \beta M_\tau^E d\tau}$$

As $Q_T \in (0,1)$ only for a finite number of times and for $\epsilon > 0$ small enough we must have $Q_{T-\epsilon} = 1$, we have that the payoff from applying at T is strictly lower than the payoff from applying at $T-\epsilon$, by continuity of μ_t:

$$e^{-rT}(\mu_{T-\epsilon}v-(1-\mu_{T-\epsilon}))e^{-\int_0^{T-\epsilon} \beta M_\tau^E d\tau}$$

(ii) Otherwise, $\eta(i|a_T^i = 1) \le S_T$. If the inequality is strict, by the definition of T, some agent decides never to apply for available units of the good, even though it is profitable to do so at time T. If it holds with equality, as $\bar{M} < 1$, some agents never get the good, and get a payoff of 0. However, by $v > v_0^M$, applying at time 0 is profitable: as $T > 0$, the good is available at time 0.

Given that these two cases contradict equilibrium behavior, we conclude that we must have $T = 0$, and all agents apply at time 0.

A.8 Proposition 6 (Simple Plans)

The first step is to show that simple plans always induce equilibria. The next step is to show that these are optimal.

The fact that simple plans exist can be trivially observed by noting that $S_t = 0$ for every t would generate no application to adopt as an equilibrium. If

For the fist objective, note that one needs only to find times and batch masses guaranteeing the incentive compatibility constraints. Given exhaustion at the time of release, there is no need to check for behavior between batches.

Formally, given release batches at points $(t_1, t_2, ..., t_n)$, application of a type v^n to adopt at time t_i implies that $t_i \succeq^{v^n} t_j$ for any $j \ne i$.

This

I will now show that focusing on simple supply plans is without loss of generality. Two steps will be done for this: 1) there is an optimal supply path $\{S_t\}$ that induces immediate exhaustion of batches (a mass of at least the size of the batch applies to get the good). 2) There is an optimal supply plan among this class with at most N jumps.

Step 1: Immediate Exhaustion

Take a plan $\{S_t\}$ for which there is no immediate exhaustion for some batch at a time t'. We will show that there is another plan $\{S_t'\}$ with immediate exhaustion at time t' that induces an adoption path that the principal weakly prefers.

Without immediate exhaustion and the fact that the principal must choose supply plans in batches, we have free supply with increasing M_t for some interval (t', t''). From Lemma 8 and Proposition 1, we have that agents of a single type must adopt at any moment, and the induced adoption path $\{M_t\}$ must increase convexly in this interval.

Suppose that no batch is released after time t' first. Then, the principal can do strictly better by releasing one last batch later and have the lowest types adopting myopically.

If another batch is released at a time $T > t'$, it must be the case that for some type v^n applying at t', that $t' \succeq^{v^n} T$. Take the mass of agents adopting on the interval (t', t''), and denote it by m^*. The principal can then release exactly the amount on t' and add the mass m^* and the following batch at a time $T' < T$ such that $t' \sim^{v^n} T'$.

It is easy to see why the principal would weakly prefer that: it induces the same mass of adoption up to time T' but potentially earlier.

Step 2: Up to N batches

This will come directly from the fact that agents from a particular type must adopt at most two different points in time and that if they do so in two points, they must be adopting together with the type immediately below them in value.

To see the first point, suppose that two agents of the same type v are adopting the innovation at two points in time $t_1 < t_2$. Then $t_1 \sim^v t_2$. I will argue that any point in time $t \in (t_1, t_2)$ is strictly preferred for this agent compared to either of the two times. Note, from lemma 8, that no lower or higher value agent will take up between these points. But then the agent will have the following payoff between these two points:

$$e^{-rt}(v - v_{t_1}^M e^{-\beta M_{t_1}(t-t_1)})$$

From $t_1 \sim^v t_2$, we know that this value at $t = t_1$ equals the value at $t = t_2$. The derivative of this value with respect to t is positive at t_1, as this value has a unique maximizer, which finishes the argument.

A.9 Proposition 7

Denote by T^F the time in which type v^n adopts myopically for $\{M_t^F\}$, the equilibrium with free-supply.

Let's compare the equilibria induced by $S_t = \bar{M}$ for every t with $\{M_t^F\}$. Let's first guess and then verify that this restricted equilibrium adoption path $\{M_t^R\}$ is as follows:

$$\{M_t^R\} = \begin{cases} M_t^F \text{ for } t < T^F \\ \bar{M} \text{ for } t \geq T^F \end{cases}$$

Type v^1 agents clearly have no incentive to deviate. Given that, types v^2 agents also do not have incentives. The only type that requires closer examination is type v^n. For $\{M_t^R\}$, it must be the case that all agents of that type apply at T^F. They get a payoff of 0. If they do not apply, they get a payoff of 0 as well. As the principal can induce its favorite equilibrium, they all apply at this point in time.

Clearly this benefits the principal, when compared to free supply, and therefere she is strictly better-off with supply restrictions.

A.10 Theorem 2

The fact that there is an optimal supply plan with up to two batches is a conclusion from Proposition 6. We must show that the supply plan described is indeed optimal among simple plans.

Note first that the time the first batch is released must be 0. To see that, note that otherwise, the game is the same from 0 to t_1, and the principal is strictly worse off.

There are, then, three variables to choose from:

1. The mass m_1 to be released at time 0,
2. When to release the second batch, t_2,
3. The mass m_2 to be released at the second batch

One can see that an optimal m_2 equals $1-m_1$. We need to determine, then, t_2 and m_1.

Note also that it is optimal for the principal to have the type v^2 agents to adopt *myopically*. This is the case because they cannot adopt before that, and anytime after is just decreasing the payoff for the principal. Therefore we establish that t_2 will be such that the payoff of v^2 is 0.

If $\bar{T}_1^D(q_1) < T_2^M(q_1)$, we have that one should set the supply plan as stated. Otherwise, we must serve a mass m_1^* at time 0, the lowest point in which a type v^1 is indifferent between pickup at this time and 0.

This concludes the proof.

A.11 Theorem 3

The proof of the theorem will be done through a series of steps, following the intuitive discussion done in the main text:

STEP 1: If $\bar{T}_1^D(q_1) > T_3^M(q_1,0,0)$, the optimal supply-plan has two batches. One at time 0 serving up to m_1^* and the second at $T_3^M(m_1^*,0,0)$, serving $1-m_1^*$.

Suppose the principal prefers another plan $\{S_t'\}_t$. By definition, then, we need to have its induced adoption path $\{M_t'\}$ being such that $M_{T'}' = \bar{M}$ at some $T' < T_3^M(m_1^*,0,0)$. As, by definition, agents of type v^3 must prefer adopting at time T' instead of never, we must have $\int_0^{T'} M_\tau' d\tau = \int_0^T M_\tau d\tau$. This would imply, though, that $T' \succ^{v^1} 0$, as we have the same "learning" happening earlier. But then we would not have positive take-up at time 0. We can see that we have $T' \succ^{v^1} 0$ from the fact that $0 \succeq^{v^1} T_3^M(m_1^*,0,0)$ and that :

STEP 2: Check if $T_2^E(q_1) > T_3^M(q_1,0,0)$. If so, release batches at times 0 and $T_3^M(q_1,0,0)$, with $m_1 = q_1$ and $m_2 = 1-q_1$. Otherwise, continue.

The only possible deviation would be for a new mid-batch release by the principal, focused on the adoption of agents of type v^2, which right now happens at $T_3^M(q_1,0,0)$, together with the agents that are of type v^3, adopting myopically. I will now show that $T_2^E(q_1) > T_3^M(q_1,0,0)$ implies that no such batch is feasible.

From the value function of the v^2 types and the definition of $T_2^E(q_1)$, we know an agent of this type prefers adoption at this point. This means that adoption at time $T_3^M(q_1,0,0)$ is preferable to adoption at any point before, and the argument concludes.

STEP 3: Check if $\bar{T}_1^D(q_1) \prec^{v^2} T_3^M(q_1,0,0)$. If so, no batching for the v^2 types is profitable. Otherwise, proceed.

The logic is similar to the one for the step above: no mid-batch would be profitable.

STEP 4: If $T_3^M(q_1,T_1^D(q_1),q_2) \succ^{v^2} T_1^D(q_1) \succ^{v^2} T_3^M(q_1,0,0)$, there are two options: If $T_2^E(q_1) > T_1^D(q_1)$, go for the former. Otherwise, go for m_2^* at $\bar{T}_1^D$ making v^2 indifferent.

This step has the most important point for this section: it is better to wait and have more v^2-type agents adopting in a mid-batch than to have earlier adoption happening. We must explicitly lay out the principal's problem to prove this point.

The planner wants to choose a time T_2 and masses realized m_1 and m_2, at times T_1 and T_2, respectively, to minimize $T_3^M(m_1,T_2,m_2)$, the point in time in which the v^3 types want to adopt myopically. The problem of the principal is, then, to minimize $T_3^M(m_1,T_2,m_2)$ subject to $T_2 \succeq^{v^2} T_3^M(m_1,T_2,m_2)$, which leads to the following inequality:

$$v^2 - v_{T_2}^M \geq e^{-r(T_3^M(m_1,T_2,m_2)-T_2)}\left(v^2 - v_{T_2}^M e^{-\beta(m1+m_2)(T_3^M(m_1,T_2,m_2)-T_2)}\right)$$

I first argue that the inequality above must bind in this case: otherwise, we could either release the second batch at the same time and increase m_2 or keep m_2 and decrease T_2, both profitable deviations. Given that, we can isolate $T_3^M(m_1,T_2,m_2)$ from the equality constraint and get:

$$T_3^M(m_1,T_2,m_2) = \frac{1}{r}[ln(v^2 - v^3) - ln(v^2 - v_0^M e^{-rm_1T_2}]$$

We have that the FOC and SOC guarantee that $T_2 = T_2^E(q_1)$ minimizes this expression, proving the result.

STEP 5: If $\bar{T}_1^D(q_1) \succ^{v^2} T_3^M(q_1,T_1^D(q_1),q_2)$, then pick m_1^* such that $\bar{T}_1^D(m_1^*) \sim^{v^2} T_M^3(m_1^*,T_1^D(m_1^*),q_2)$. Compare this to $T_2^E(q_1)$. One of these two is optimal.

In this case, we might need "too much" time to secure a few v^1 types. If a substantial mass of v^2 types is relatively easy to convince, the principal might be better off with partial adoption at 0. This would roughly correspond to the case in which $q_2 >> q_1$ (many more mid-types) and $v^2 \sim v_0^M$ (mid-types are "easy" to persuade).

The concavity of the function to be minimized guarantees that the minimum must be one of the corner solutions, giving us the result.

A.12 Proposition 8

We can analyze the continuous economy directly from Proposition 3. I will show that a positive mass of agents for $v \sim F$ would want to adopt together with $\underline{v}$ first.

At time T of a release of the last batch, $\underline{v}$ adopts myopically in any optimal supply plan. Take the type v^E who would want to adopt strategically at T. It must be, then, $v^E \equiv \underline{v}\left(\frac{\beta}{r}+1\right) > \underline{v}$. By full support, there is a positive mass of agents with types in this interval. To see that they would all want to adopt with type $\underline{v}$, note that any plan that has this type adopting at a time T and any type in this interval adopting strictly before T would break their incentive compatibility constraints, as they would rather wait and adopt at time T (as this is their equilibrium adoption time).

B Examples

B.1 Example of optimal supply plan with 3 batches

Suppose that we have valuations for the three types given by $(v^1, v^2, v^3) = (1.2, 0.8, 0.5)$, a mass of each type given by $(q_1, q_2, q_3) = (0.5, 0.3, 0.2)$, $\beta = r = 1$ and $\mu_0 = 0.5$, so that $v_0^M = 1$.

Then $T_3^M(q_1, 0, 0) = 2ln(2) \sim 1.39$, $\bar{T}_1^D(q_1) = 1.16587$, so we have that $\bar{T}_1^D(q_1) < T_3^M(q_1, 0, 0)$.

What about $T_2^E(q_1)$? Then one is maximizing $e^{-t}(0.8 - e^{-q_1 t})$, so that $T_2^E(q_1) = 1.2572 < T_3^M(q_1, 0, 0)$, so we are also good here.

Finally, we also have that $\bar{T}_1^D(q_1) \succ^{v^2} T_3^M(q_1, 0, 0)$, from the equations determining payoffs.

Together, these equations mean that mid-value agents are willing to take up at a time when high-types no longer want to and can, therefore, only speed up learning for low-types. The $\bar{T}_1^D(q_1) \succ^{v^2} T_3^M(0, q_1)$ condition guarantees that learning is not such that the mid-level agent would rather wait and take up with the lowest types.

B.2 Examples of ambiguous welfare outcome for mid-type

Let's first focus on one case in which the optimal plan leads to a worse outcome for the mid-type. Take $v^{\mathcal{N}} = (1.2, 0.50001, 0.5)$ and $q^{\mathcal{N}} = (0.6, 0.1, 0.3)$. We also have $\beta = r = v_0^M = 1$. We have that $\bar{T}_1^D(q_1) > T_3^M(q_1)$, and therefore it is optimal to use two batches only. In this case, the payoff for type v^2 is close to 0 the payoff for the v^3 type. With free supply, we have the $M_0^F = 0.2$. If only this mass adopted at time 0 (a case worse than the one with free supply), type v^2 would adopt at time ~ 4.3 only and achieve a payoff higher than 0.001.

Now, let's consider a case in which the optimal plan leads to a better outcome for the mid-type. Take $v^{\mathcal{N}} = (1.2, 0.8, 0.5)$ and $q^{\mathcal{N}} = (0.3, 0.5, 0.2)$. We have $\beta = r = v_0^M = 1$. In this case $\bar{T}_1^D(q_1) < T_2^M(q_1)$. It is also the case that not all of v^2 types can be captured in a mid-batch. From the optimal algorithm for three types presented above, we have that the second batch will be released at time $T_2^E(q_1)$. The initial mass of adopters with free supply is equal to $M_0^F = 0.2$. Therefore, v^2 is better off with supply restrictions in this case.